وزارة التعليم العالي والبحث العلمي

المعهد التقني / السماوة

قسم المساحة

اسم المادة : **Mathematics and Spherical Triangles**

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Derivative of Trigonometric Functions

Derivative applications

Trigonometric Functions

In trigonometry, circular functions are also referred to as trigonometric functions. The definition of these functions in its simplest form is that it exhibits a close relationship between sides and angles of a triangle. Also known as trigonometric ratios, they are designated by cosecant, secant, cotangent, tangent, cosine and sine.

Derivatives of Trigonometric Functions

The table below summarizes the derivatives of 6 basic trigonometric functions:

Derivatives of all trig functions

$$\frac{d}{dx}\sin x = \cos x \qquad \frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\tan x = \sec^2 x \qquad \frac{d}{dx}\cot x = -\csc^2 x$$

$$\frac{d}{dx}\sec x = \sec x \tan x \qquad \frac{d}{dx}\csc x = -\csc x \cot x$$

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Example : Find the derivative of the following function.

Derivatives of Trig Functions: Selected Problems

1. Find $f'(x)$:

(a) $f(x) = 4\cos x + 2\sin x$

$$f'(x) = 4\frac{d}{dx}(\cos x) + 2\frac{d}{dx}(\sin x)$$
$$= \boxed{-4\sin x + 2\cos x}$$

(b)

$$\left(3x + \sin(x) - 4\cos(x)\right)' = \left(3x\right)' + \left(\sin x\right)' - \left(4\cos x\right)' =$$
$$= 3x' + \cos x - 4\left(\cos x\right)' =$$
$$= 3\cdot 1 + \cos x - 4\cdot\left(-\sin x\right) =$$
$$= 3 + \cos x + 4\sin x$$

Example : Find the derivative of the following function.

$$\boldsymbol{g(x) = secx + cscx}$$

Solution:

$$\boldsymbol{\frac{d}{dx}g(x) = \frac{d}{dx}secx + \frac{d}{dx}cscx}$$
$$\boldsymbol{= secx\ tanx - cscx\ cotx}$$

Example : Find the derivative of the following function.

$$\boldsymbol{g(x) = secx \cdot (1 + sinx)}$$

Solution:

$$\frac{d}{dx}g(x) = \frac{d}{dx}secx \cdot (1 + sinx) + secx \cdot \frac{d}{dx}(1 + sinx)$$

$$= (secx \cdot tanx) \cdot (1 + sinx) + secx \cdot cosx$$

Example : Find the derivative of the following function.

$$g(x) = \frac{x}{cosx}$$

Solution:

$$\frac{d}{dx}g(x) = \frac{\frac{d}{dx}x \cdot cosx - x \cdot \frac{d}{dx}cosx}{cos^2x}$$

$$= \frac{1 \cdot cosx + x \cdot sinx}{cos^2x} = \frac{cosx + x \cdot sinx}{cos^2x}$$

Derivative applications

The derivative application is used as :

ds/dt = v , dv/dt = a

Where:

S : the distance I meter (m)

V: the velocity in (m/s)

A: the acceleration (m/s2)

Example 1: The position of a particle on a line is given by $s(t) = t^3 - 3t^2 - 6t + 5$, where t is measured in seconds and s is measured in feet. Find

a. The velocity of the particle at the end of 2 seconds.

b. The acceleration of the particle at the end of 2 seconds.

Part (a): The velocity of the particle is

$$v = s'(t) = 3t^2 - 6t - 6$$

At $t = 2$ seconds $s'(2) = 3(2)^2 - 6(2) - 6$

$s'(2) = -6$ ft/sec

Part (b): The acceleration of the particle is

$$a = v'(t) = s''(t) = 6t - 6$$

At $t = 2$ seconds $v'(2) = s''(2) = 6(2) - 6$

$v'(2) = s''(2) = 6$ ft/sec^2

Example 2: The formula $s(t) = -4.9t^2 + 49t + 15$ gives the height in meters of an object after it is thrown vertically upward from a point 15 meters above the ground at a velocity of 49 m/sec. How high above the ground will the object reach?

The velocity of the object will be zero at its highest point above the ground. That is, $v = s'(t) = 0$, where

$$v = s'(t) = -9.8t + 49$$

$$s'(t) = 0 \Rightarrow -9.8t + 49 = 0$$

$$-9.8t = -49$$

$$t = 5 \text{ seconds}$$

The height above the ground at 5 seconds is

$$s(5) = -4.9(5)^2 + 49(5) + 15$$

$$s(5) = 137.5 \text{ meters}$$

Homework

Find the derivative of the following function.

1 - $y = x^2 (3x+1)$

2 - $f(x) = tanx + cotx$

3 - $h(x) = cscx + sinx \cdot tanx$