وزارة التعليم العالي والبحث العلمي

المعهد التقني / السماوة

المساحة

اسم المادة : Mathematics and Spherical Triangles

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❖ Equation of the straight line

Equation of the straight line

The equation of a straight line is usually written this way:

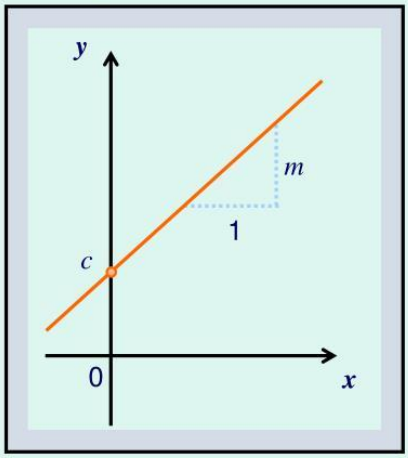
$$y = mx + b$$

(or "y = mx + c")

The equation of a straight line

The equation of a straight line can be written in several forms.

You are probably most familiar with the equation written in the form $y = mx + c$.



The value of m tells us the **gradient** of the line.

The value of c tells us where the line cuts the y-axis.

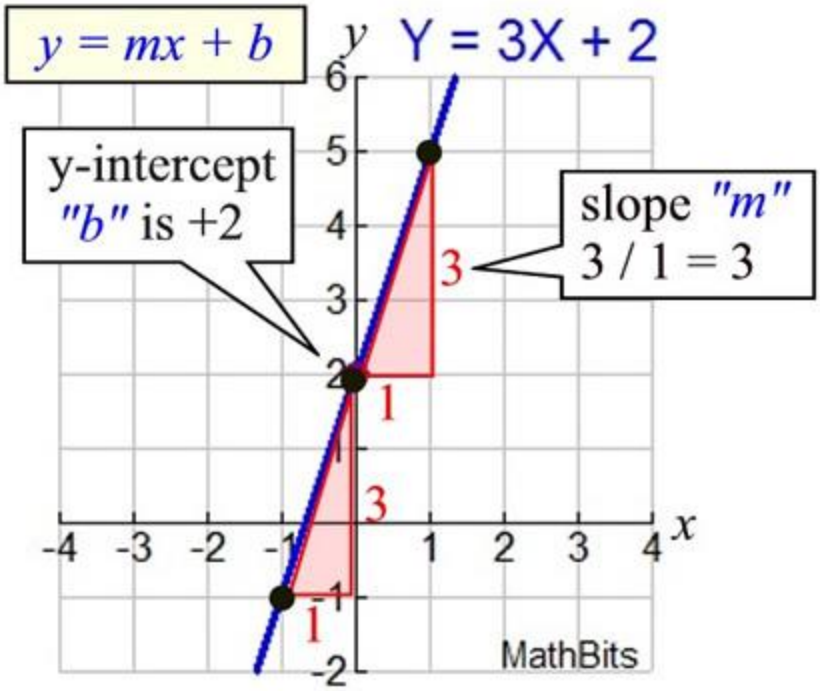
This is called the ***y*-intercept** and it has the coordinates $(0, c)$.

For example, the line $y = 3x + 4$ has a gradient of **3** and crosses the y-axis at the point (0, **4**).

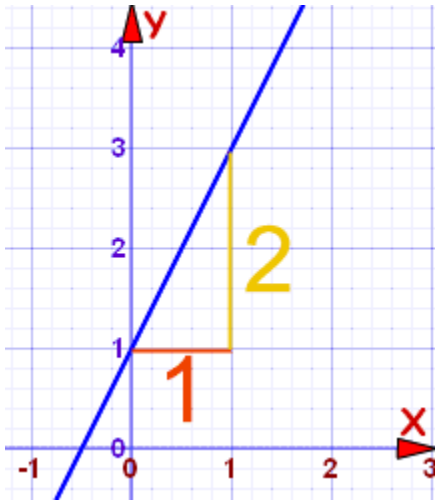
m = (Change in Y / Change in X)

Knowing this we can work out the equation of a straight line:

Example 1:Find the equation of the straight line.



Example 2: Find the equation of the straight line .



m = *2/**1*** = 2

b = 1 *(value of **y** when x=0)*

So: y = 2x + 1

With that equation you can now ...

... choose any value for x and find the matching value for y

For example, when x is 1:

$$y = 2\times\mathbf{1} + 1 = 3$$

Check for yourself that x=1 and y=3 is actually on the line.

Or we could choose another value for x, such as 7:

$$y = 2\times\mathbf{7} + 1 = 15$$

And so when x=7 you will have y=15

Parallel and Perpendicular Lines

Parallel Lines

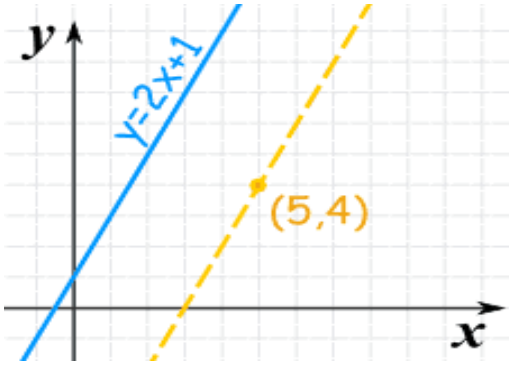
Suppose two lines are parallel. Then the angle between them must be 0. That is, $\theta=0$, which makes $\tan\theta = 0$.

In other words, the slopes of the two parallel lines must be equal $\mathbf{m_1 = m_2}$. This seems obvious, as two parallel lines must make the same angle with a transversal (i.e. the X-axis)

Suppose the lines $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ are parallel.

Then the slopes of these lines must be equal. Therefore, we have $-a_1/b_1 = -a_2/b_2$ or $\mathbf{a_1/a_2 = b_1/b_2}$

Example1: Find the equation of the line that is:



- parallel to **y = 2x + 1**
- and passes through the point (5,4)

Solution:

The slope of **y=2x+1** is: 2

The parallel line needs to have the same slope of 2.

We can solve it using the "point-slope" equation of a line:

$$y - y_1 = 2(x - x_1)$$

And then put in the point (5,4):

$$y - 4 = 2(x - 5)$$

And that answer is OK, but let's also put it in y = mx + b form:

$$y - 4 = 2x - 10$$

$$y = 2x - 6$$

Be careful! They may be the **same line** (but with a different equation), and so are **not parallel**.

How do we know if they are really the same line? **Check their y-intercepts** (where they cross the y-axis) as well as their slope:

Example: is $y = 3x + 2$ parallel to $y - 2 = 3x$?

Solution:

For $\mathbf{y = 3x + 2}$: the slope is 3, and y-intercept is 2

For $\mathbf{y - 2 = 3x}$: the slope is 3, and y-intercept is 2

In fact they are the same line and so are not parallel.

Perpendicular Lines

In this case $\theta = 90°$, or $\cot \theta = 0$.
This leads us to $\mathbf{m_1 m_2 = -1}$. That is, the product of the slopes of two perpendicular lines must be equal to -1

Conversely, if the product of the slopes of two lines equals -1, then the lines must be perpendicular.

And, analogous to the previous result, if the lines $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ are perpendicular, then $-a_1/b_1 \times -a_2/b_2 = -1$,or $\mathbf{a_1a_2 + b_1b_2 = 0}$

That's it for now. I'll talk a bit more about equations of parallel and perpendicular lines while covering <u>examples</u> (so don't miss them).

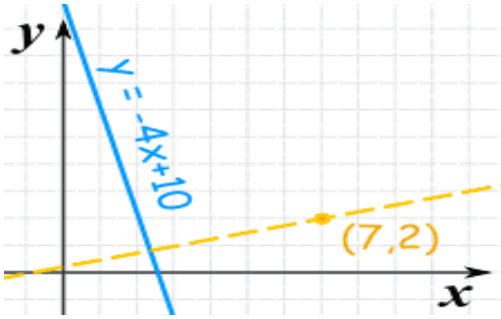
Two lines are Perpendicular when they meet at a right angle (90°).

To find a perpendicular slope:

When one line has a slope of m, a perpendicular line has a slope of $-1/\mathbf{m}$

In other words the **negative <u>reciprocal</u>**

Example 1: Find the equation of the line that is



- perpendicular to **y = −4x + 10**
- and passes through the point **(7,2)**

Solution:

The slope of **y=−4x+10** is: −4

The **negative reciprocal** of that slope is:

$$m = -1/-4 = 1/4$$

So the perpendicular line will have a slope of 1/4:

$$y - y_1 = (1/4)(x - x_1)$$

And now put in the point (7,2):

$$y - 2 = (1/4)(x - 7)$$

And that answer is OK, but let's also put it in "y= mx +b" form:

$$y - 2 = x/4 - 7/4$$

$$y = x/4 + 1/4$$

Example 2:

Find the equation of a straight line that is perpendicular to y=2x+1.

Solution:

$\boldsymbol{y = 2x + 1}$ The gradient of $\boldsymbol{y = 2x + 1}$ **is 2**

To find the perpendicular gradient, find the number which will multiply by 2 to give -1. This is the negative reciprocal of the gradient.

The reciprocal of 2 is $\frac{1}{2}$, so the negative reciprocal of 2 is $-\frac{1}{2}$

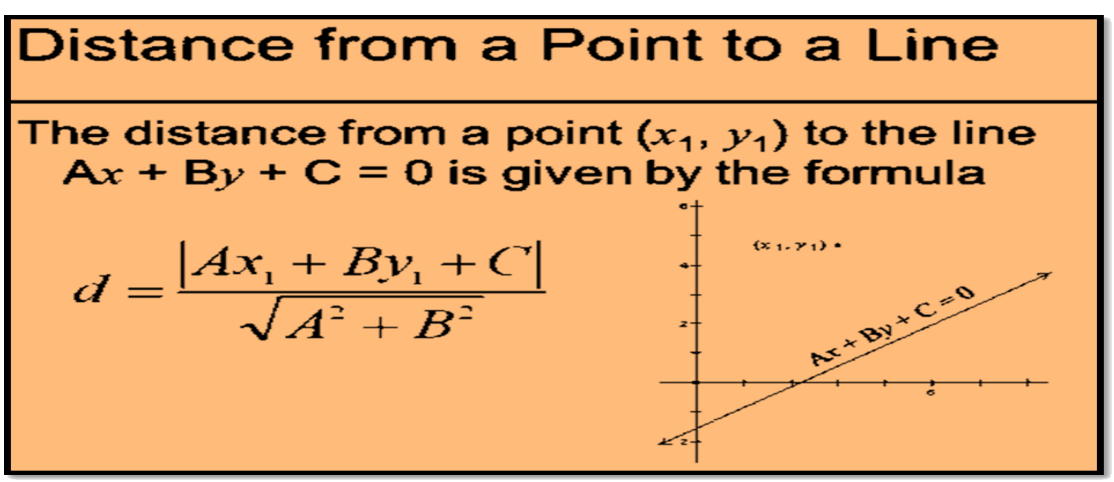
This gives $\boldsymbol{y = -\frac{1}{2}x + c}$

Lesson Summary

1. Two lines, whose slopes are m_1 and m_2 are parallel if $m_1 = m_2$
2. Two lines, whose slopes are m_1 and m_2 are perpendicular if $m_1.m_2 = -1$
3. Two lines, whose equations are $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ parallel if $a_1/a_2 = b_1/b_2$
4. Two lines, whose equations are $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ perpendicular if $a_1a_2 + b_1b_2 = 0$.

Distance between a point and a straight line

The distance between a point P and a straight line r is the minimum of the distances between P and any point on the straight line.



Let Ax+By+C=0 be the general equation of the straight line r, and P=(p_1,p_2) the given point and A=(a_1, a_2)any point of the straight line.

If we take a perpendicular vector to r, for example n=(A,B) for the properties of the scalar product in the vectors projection we have:

$$\boxed{d(P,r) = \frac{|A \cdot p_1 + B \cdot p_2 + C|}{\sqrt{A^2 + B^2}}}$$

Example:

Let P=(−1,2) be a point and r: 4x−3y+1 = 0 a straight line. Calculate the distance between the point and the straight line.

Applying the previous formula we have:

$$d(P,r) = \frac{A \cdot p_1 + B \cdot p_2 + C}{\sqrt{A^2 + B^2}} = \frac{|4 \cdot (-1) + (-3) \cdot 2 + 1|}{\sqrt{4^2 + (-3)^2}} = \frac{9}{5}$$

Distance Between 2 Points

Formula for Distance Between Two Points

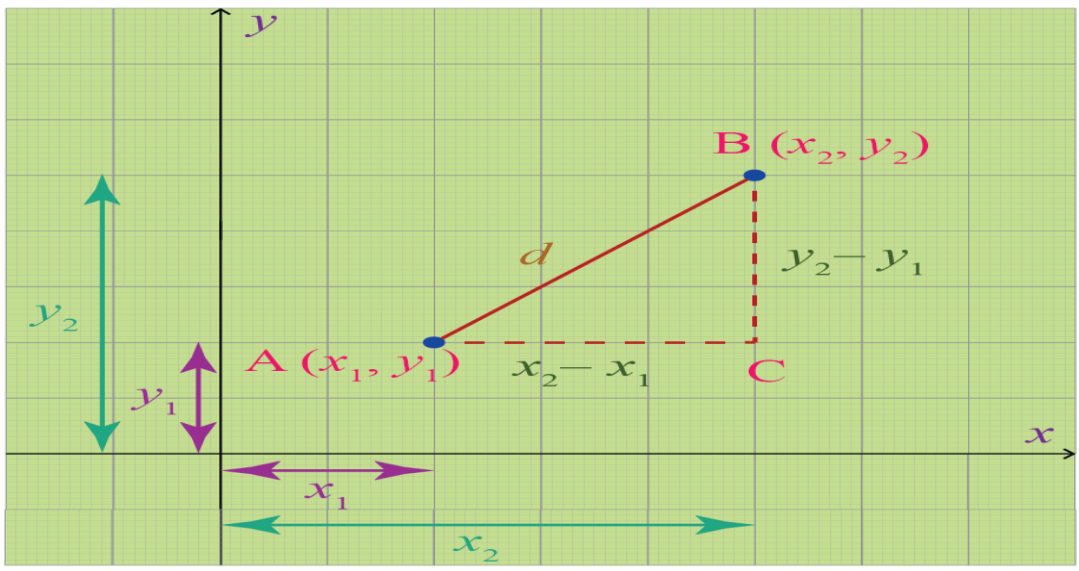
The formula for the distance, d, between two points whose coordinates are (x_1,y_1) and (x_2,y_2) is:

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

This is called the **Distance Formula.**

Let's learn how to derive this formula next

Next, we will construct a right-angled triangle with $\overline{AB}$ as the hypotenuse.



The distance, d **, between two points whose coordinates are** (x_1,y_1) **and** (x_2,y_2) **is:**

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Here, we have used the absolute value signs because distance can never be negative.

Example 1

Find the distance between the two points $(2,-6)$ and $(7,3)$

Solution:

Let us assume the given points to be:

$(x_1,y_1)=(2,-6)$

$(x_2,y_2)=(7,3)$

The formula to find the distance between two points is:

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

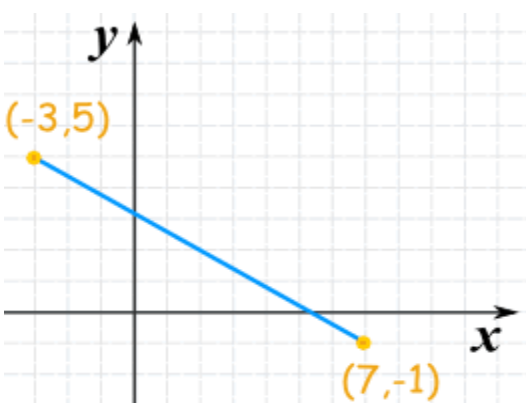
$d=\sqrt{(7-2)^2+(3-(-6))^2}$

$d=\sqrt{(5)^2+(9)^2}$

$d=\sqrt{25+81}$

$d=\sqrt{106}$

Example 2: Find the distance between the two points (7,−1) and (-3,5).



Fill in the values: $c=\sqrt{(-3-7)^2+(5-(-1))^2}$

$$c=\sqrt{(-10)^2+(6)^2}=\sqrt{100+36}=11.66...$$

(Note √136 can be further simplified to 2√34 if you want)