وزارة التعليم العالي والبحث العلمي
المعهد التقني/ السماوة
قسم المساحة

اسم المادة : Mathematics and Spherical Triangles

اسم المدرسة :م . م لقاء طارق هادي

- Solving Triangles
- Law of Sines
- Law of Cosines

Solving Triangles

"Solving" means finding missing sides and angles.

When we know any 3 of the sides or angles ...

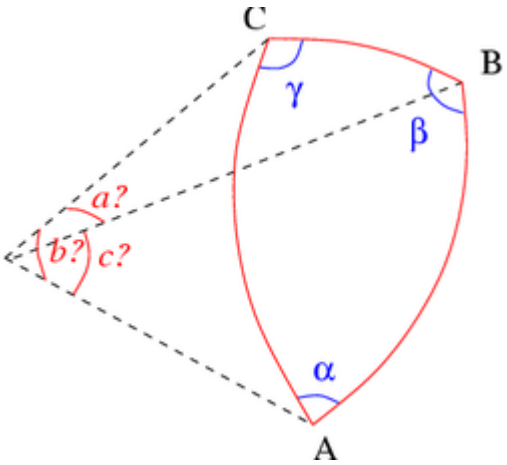
... we can find the other 3
(Except for only 3 angles, because we need at least one side to find how big the triangle is.)

Six Different Types

There are SIX different types of puzzles you may need to solve. Get familiar with them:

1- Three angles given (AAA)

This means we are given all three angles of a triangle, but no sides.

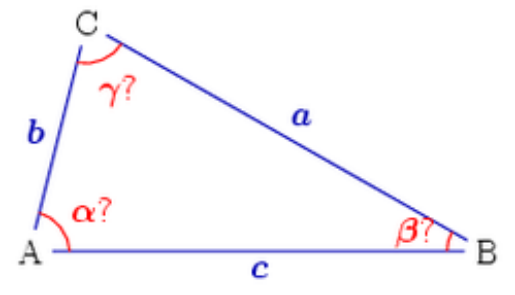


Three angles given

AAA triangles are impossible to solve further since there are is nothing to show us **size** ... we know the shape but not how big it is.

We need to know at least one side to go further.

2-Three sides given (SSS)



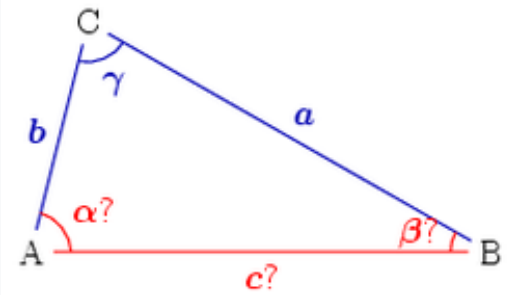
Three sides given

Let three side lengths a, b, c be specified. To find the angles α, β, the law of cosines can be used:

Then angle $\boldsymbol{\gamma = 180° - \alpha - \beta}$.

We can find angle β from the law of sines .

3-Two sides and the included angle given (SAS)



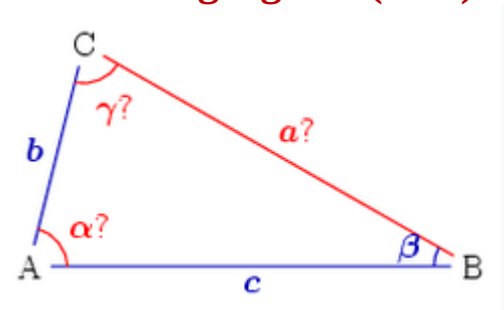
Two sides and the included angle given

Here the lengths of sides a, b and the angle γ between these sides are known. The third side can be determined from the law of cosines:

Now we use law of cosines to find the second angle:

Finally, $\boldsymbol{\beta = 180° - \alpha - \gamma}$.

4-Two sides and non-included angle given (SSA)



Two sides and a non-included angle given

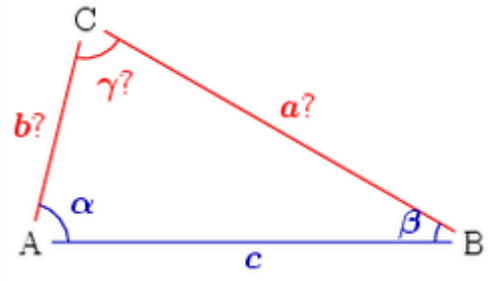
Assume that two sides b, c and the angle β are known. The equation for the angle γ can be implied from the law of sines:

Once γ is obtained, the third angle $\boldsymbol{\alpha = 180° - \beta - \gamma}$.

The third side can then be found from the law of sines:

or from the law of cosines:

5- A side and two adjacent angles given (ASA)



One side and two adjacent angles given

The known characteristics are the side c and the angles α, β. The third angle $\boldsymbol{\gamma = 180° - \alpha - \beta}$.

Two unknown sides can be calculated from the law of sines:

6-A side, one adjacent angle and the opposite angle given (AAS)

The procedure for solving an AAS triangle is same as that for an ASA triangle: First, find the third angle by using the angle sum property of a triangle, then find the other two sides using the law of sines.

Solving All Types of Triangles

If you know this...		**You can solve the triangle this way...**
three angles, **AAA**		There's not enough information. Without at least one side you have the shape of the triangle, but no way to scale it correctly.
two angles and a side, **AAS** or **ASA**		Find the third angle by subtracting from 180°. Then use the Law of Sines ★ twice to find the second and third sides.
two sides and ...	the included angle, **SAS**	Use the Law of Cosines ★ to find the third side. Then use either the Law of Sines ★ or the Law of Cosines ★ to find the second angle.
	a non-included angle, **SSA**	Use the Law of Sines ★ to get the second angle, and the Law of Sines ★ to get the third side.
three sides, **SSS**		Find one angle with the Law of Cosines . Use that angle and its opposite side in the Law of Sines to find the second angle, then subtract to find the third angle.

★ If a 90° angle is given, the Law of Sines and the Law of Cosines are overkill. Just apply the definitions of the sine and cosine and the tangent to find the other sides and angles.

Here is some simple advice:

- When the triangle has a right angle, then use it, that is usually much simpler.
- When two angles are known, work out the third using Angles of a Triangle Add to 180°.
- Try The Law of Sines before the The Law of Cosines as it is easier to use.

Summary: A triangle has six parts, **three sides and three angles**. Given almost any three of them—three sides, two sides and an angle, or one side and two angles—you can find the other three values. This is called **solving the triangle**, and you can do it with **any triangle, not just a right triangle**.

For all of this, you need only two tools, the Law of Sines and the Law of Cosines. The Law of Sines relates any two sides and the angles opposite them, and the Law of Cosines relates all three sides and one angle.

Law of Sines

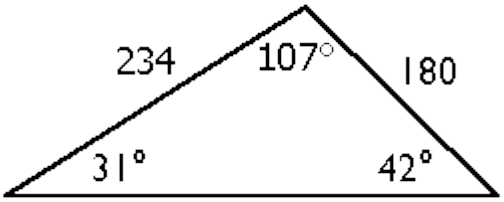
Law of Sines—First Form:

$a/\sin A = b/\sin B = c/\sin C$

for any triangle, if you **divide any side by the sine of the opposite angle**, you'll get the same result. This law is valid for any triangle.

Coming back to our original triangle, we can compute the length of the third side:

$a/\sin A = c/\sin C$
$a\,(\sin C)/(\sin A) = c$



$c = 180 \times (\sin 107°)/(\sin 31°) \approx 334$

Law of Sines—Second Form:

$(\sin A)/a = (\sin B)/b = (\sin C)/c.$

In most cases where you use the Law of Sines, you get a unique solution. But sometimes you get two solutions (or none) in the side-side-angle case, where you know two sides and an angle that's not between them.

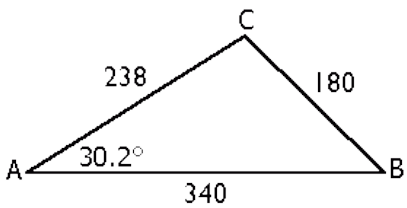
Law of Cosines

Law of Cosines—First Form:

$\cos A = (b^2 + c^2 - a^2) / 2bc$

$\cos B = (a^2 + c^2 - b^2) / 2ac$

$\cos C = (a^2 + b^2 - c^2) / 2ab$



let's find the other two angles of that triangle:

$\cos C = (a^2 + b^2 - c^2) / 2ab$
$\cos C = (180^2 + 238^2 - 340^2) / (2 \times 180 \times 238)$
$\cos C \approx -0.309944$
$C \approx 108.1°$

Law of Cosines—Second Form:

$$a^2 = b^2 + c^2 - 2bc \times \cos A$$

$$b^2 = a^2 + c^2 - 2ac \times \cos B$$

$$c^2 = a^2 + b^2 - 2ab \times \cos C$$

Typically you'll use the Law of Cosines in **the first form for finding an angle and the second form for finding a side**.

Probably you don't want to try to remember that, but it's not as hard as it looks. I think of it this way: the square of one side is the sum of the squares of the other two, like Pythagoras, but with a "correction factor" of 2 times those same sides times the cosine of the opposite angle.