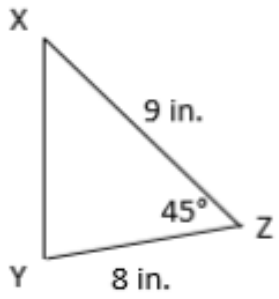
وزارة التعليم العالي والبحث العلمي
المعهد التقني/ السماوة
قسم المساحة

اسم المادة : Mathematics and Spherical Triangles

اسم المدرسة : م . م لقاء طارق هادي

(Various exercises to solve the triangle)

Example :Solve this triangle



We need to find $m\measuredangle X$, $m\measuredangle Y$, and the length of side z. Let's take them one at a time.

1) First we'll use the Law of Cosines to find the length of side z.

We know we need to use the Law of Cosines because we are have SAS (two sides and the angle in between.)

C will be 45°. a and b are 9 and 8. Let's substitute and solve:

$c^2 = 9^2 + 8^2 - 2(9)(8)\cos 45°$

$c^2 = 81 + 64 - 144\cos 45°$

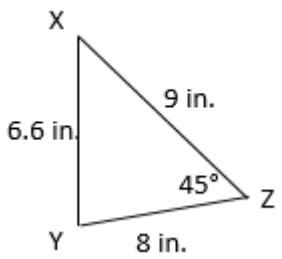
$c^2 = 145 - 144\cos 45°$

$c = \sqrt{145 - 144\cos 45°}$

$c \approx 6.6$

2) Now we need to find $m\measuredangle X$.

Let's input our new info into our triangle:



We can use either the Law of Sines or the Law of Cosines. Let's use the

Law of Sines (rewritten to portray angles X, Y, and Z instead of A, B, and C)

$$\frac{x}{\sin X} = \frac{y}{\sin Y} = \frac{z}{\sin Z}$$

$$\frac{8}{\sin X} = \frac{9}{\sin Y} = \frac{6.6}{\sin 45°}$$

$$\frac{8}{\sin X} = \frac{6.6}{\sin 45°}$$

$$8(\sin 45°) = 6.6 \cdot \sin X$$

$$\sin X = \frac{8(\sin 45°)}{6.6}$$

$$m\measuredangle X \approx 59°$$

3) Find $m\measuredangle Y$

Since we know that the three angles of a triangle add up to 180°, we can subtract the two known angles to find the third:

180° - 45° - 59° = 76°

$$m\measuredangle Y \approx 76°$$

you can find $m\measuredangle Y$ using either the Law of Sines or the Law of Cosines that both answers are dependent upon your calculation of side z, and z was rounded to the tenths place.

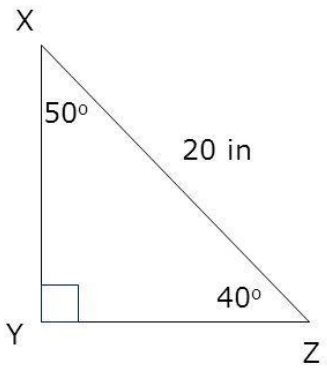
Solve a right triangle that has a 40° angle and a 20 inch hypotenuse.

Example 2

Find $m\angle X$ by using the Triangle Sum Theorem.

$180^\circ = 90^\circ + 40^\circ + m\angle X$

$50^\circ = m\angle X$



Approximate YZ by using a sine ratio.

$\sin 40^\circ = \frac{XY}{20}$

$20 \bullet \sin 40^\circ = XY$

$20 \bullet 0.6428 \approx XY$

$12.9 \approx BC$

Approximate AB by using a cosine ratio.

$\cos 40^\circ = \frac{YZ}{20}$

$20 \bullet \cos 40^\circ = YZ$

$20 \bullet 0.7660 \approx YZ$

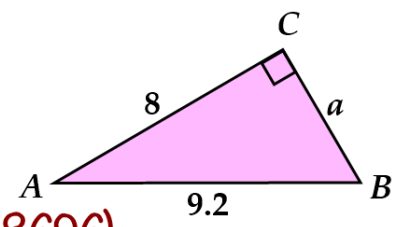
$15.3 \approx YZ$

ANSWER

The angle measures are 40°, 50°, and 90°. The side lengths are 12.9 in., about 15.3 in., and 20 in.

Example :Solve the triangle

Solve the right triangle. Round decimals to the nearest tenth.



$\cos A = \frac{8}{9.2}$

$\cos A = 0.8696$

$m\angle A = \cos^{-1}(0.8696)$

$m\angle A = 29.6^\circ$

$a^2 + 8^2 = 9.2^2$

$a^2 + 64 = 84.64$

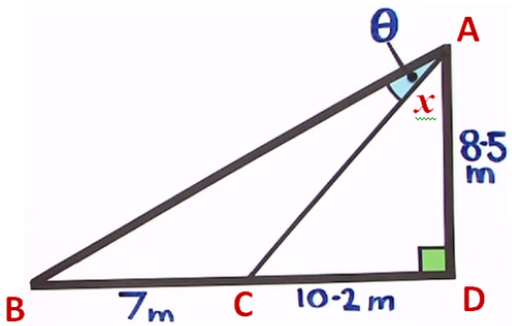
$a^2 = 20.64$

$a = 4.5$

$m\angle B = 90 - 29.6^\circ$

$m\angle B = 60.4^\circ$

Example :Find the value of the angle θ



$$\tan x = \frac{10.2}{8.5}$$

$$x = \tan^{-1}\left(\frac{10.2}{8.5}\right)$$

$$x = 50.19°$$

$$\tan A = \frac{7+10.2}{8.5}$$

$$\angle A = \tan^{-1}\left(\frac{17.2}{8.5}\right)$$

$$\angle A = 63.7°$$

$$\theta = \angle A - x$$

$$\theta = 63.7° - 50.19°$$

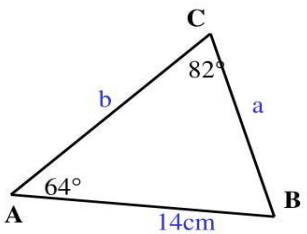
$$\theta = 13.51°$$

Example :Solve the triangle

Solving SAA or ASA Triangles

SAA and ASA Triangles – Oblique triangles in which you know 2 angles and 1 side.

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \quad \text{or} \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



Solving for <B

There are 180 degrees in a triangle, so

64 + 82 + B = 180

146 + B = 180

B = 34°

Solving for a

$$\frac{\text{Sin }(82)}{14} = \frac{\text{Sin }(64)}{a}$$

aSin(82) = 14 Sin(64)

$$a = \frac{14\ \text{Sin}(64)}{\text{Sin }(82)}$$

a = 12.7cm

Solving for b

$$\frac{\text{Sin}(82)}{14} = \frac{\text{sin}(34)}{b}$$

b = 7.9 cm

Example :Solve the triangle

Solve the right triangle. Round decimals to the nearest tenth.

$\cos 25^\circ = \frac{a}{13}$

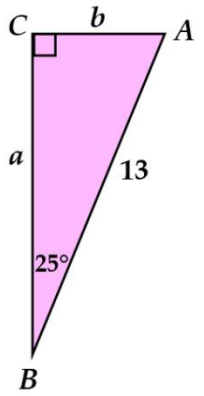
$a = 11.8$

$\sin 25^\circ = \frac{b}{13}$

$b = 5.5$

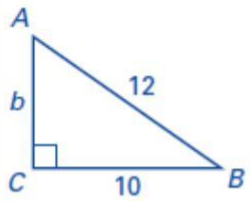
$m\angle A = 90 - 25^\circ$

$m\angle A = 65^\circ$



Example :Solve the triangle

Solve the right triangle. Round decimals to the nearest tenth.



$$12^2 = 10^2 + b^2$$

$$b = \sqrt{144 - 100} = \sqrt{44}$$

$$b \approx 6.6332 \approx 6.6$$

$$\cos B = \frac{10}{12} = \frac{5}{6}$$

$$\cos B = 0.8$$

$$B = \cos^{-1}(0.8)$$

$$B \approx 36.869^\circ \approx 36.9^\circ$$

$$m < A = 90^\circ - 36.9^\circ$$

$$m < A \approx 53.1^\circ$$

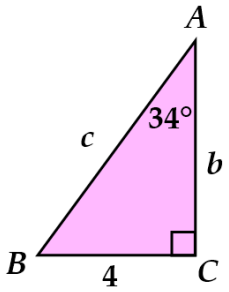
Example :Solve the triangle

Solve the right triangle. Round decimals to the nearest tenth.

$\tan 34^\circ = \frac{4}{b}$

$b \cdot \tan 34^\circ = 4$

$b = 5.9$



$4^2 + 5.9^2 = c^2$

$16 + 35.2 = c^2$

$51.2 = c^2$

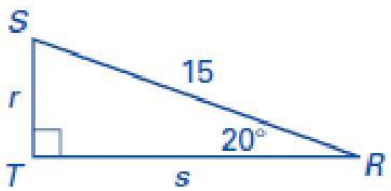
$c = 7.2$

$m\angle B = 90 - 34^\circ$

$m\angle B = 56^\circ$

Example :Solve the triangle

Solve the right triangle. Round decimals to the nearest tenth.



$< S = 90^\circ - 20^\circ$

$< S = 70^\circ$

$\sin 20^\circ = \frac{r}{15}$

$15(\sin 20^\circ) = r$

$r \approx 5.1303$

$r \approx 5.1$

$\cos 20^\circ = \frac{s}{15}$

$15(\cos 20^\circ) = s$

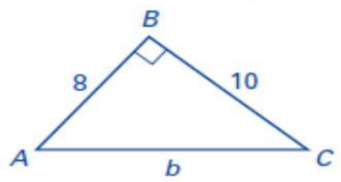
$s \approx 14.0954$

$s \approx 14.1$

Example :Solve the triangle:

But what if you don't know either of the acute angles?
To solve those triangle we must use
Inverse Trig Functions

Solve the right triangle. Round decimals to the nearest tenth.



$$\tan < A = \frac{10}{8} = 1.25$$

$$< A = \tan^{-1}(1.25)$$

$$< A = 51.3°$$

$$< B = 90° - 51.3° = 38.7°$$

$$b^2 = 8^2 + 10^2$$

$$b = \sqrt{64 + 100}$$

$$b = \sqrt{164} \approx 12.8$$