وزارة التعليم العالي والبحث العلمي

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قسم المساحة

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# Solve simultaneous equations using determinants

# 1 - Solving a System of Linear Equations Using the Inverse of a Matrix.

**With that said, here's how you find an inverse of a 2-x-2 matrix:**

**If matrix A is the 2-x-2 matrix**

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix},$$

its inverse is as follows:

$$\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Simply follow this format with any 2-x-2 matrix you're asked to find.

Armed with a system of equations and the knowledge of how to use inverse matrices, you can follow a series of simple steps to arrive at a solution to the system, again using the trusty old matrix. For instance, you can solve the system that follows by using inverse matrices:

**Example 1** : Solve the following system of linear equations by matrix inversion method:

$$\begin{cases} 4x+3y=-13 \\ -10x-2y=5 \end{cases}$$

**These steps show you the way:**

1. **Write the system as a matrix equation.**

   **When written as a matrix equation, you get (if $AX = B$, then $X = A^{-1}B$)**

$$\begin{bmatrix} 4 & 3 \\ -10 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -13 \\ 5 \end{bmatrix}$$

2. **Create the inverse of the coefficient matrix out of the matrix equation.**

   **You can use this inverse formula:**

$$\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

   **In this case, $a = 4$, $b = 3$, $c = -10$, and $d = -2$. Hence det( A) $ad - bc$ = -8+30 = 22.**

   **Hence, the inverse matrix is**

$$\frac{1}{22}\begin{bmatrix} -2 & -3 \\ 10 & 4 \end{bmatrix}$$

3. **Multiply the inverse of the coefficient matrix in the front on both sides of the equation.**

   **You now have the following equation:**

$$\frac{1}{22}\begin{bmatrix} -2 & -3 \\ 10 & 4 \end{bmatrix}\begin{bmatrix} 4 & 3 \\ -10 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{22}\begin{bmatrix} -2 & -3 \\ 10 & 4 \end{bmatrix}\begin{bmatrix} -13 \\ 5 \end{bmatrix}$$

$$= \frac{1}{22}\begin{bmatrix} (-2)(-13)+(-3)(5) \\ (10)(-13)+(4)(5) \end{bmatrix}$$

4. **Cancel the matrix on the left and multiply the matrices on the right.**

   **An inverse matrix times a matrix cancels out ($AA^{-1} = I$). You're left with**

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{22}\begin{bmatrix} 11 \\ -110 \end{bmatrix}$$

5. Multiply the scalar to solve the system.

   You finish with the *x* and *y* values:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1/2 \\ -5 \end{bmatrix}$$

**Example 2** : Solve the following system of linear equations by matrix inversion method:

$$2x + 5y = -2$$

$$x + 2y = -3$$

**Solution :**

$X = A^{-1} B$ $\quad A = \begin{pmatrix} 2 & 5 \\ 1 & 2 \end{pmatrix} \quad B = \begin{pmatrix} -2 \\ -3 \end{pmatrix} \quad X = \begin{pmatrix} x \\ y \end{pmatrix}$

$A^{-1} = (1/|A|)\ \text{adj}\ A$

$|A| = 4 - 5 = -1$

$$A = \begin{bmatrix} 2 & 5 \\ 1 & 2 \end{bmatrix} \quad B = \begin{bmatrix} -2 \\ -3 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\text{adj } A = \begin{bmatrix} 2 & -5 \\ -1 & 2 \end{bmatrix}$$

$$A^{-1} = -1/1 \begin{bmatrix} 2 & -5 \\ -1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 & 5 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} -2 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} (4 - 15) \\ (-2 + 6) \end{bmatrix} = \begin{bmatrix} -11 \\ 4 \end{bmatrix}$$

Hence the value of x and y are -11 and 4 respectively

## 2 - Cramer's Rules for Systems of Linear Equations with Two Variables

- Given a linear system

**x-column** **constant column**

$$a_1 x + b_1 y = c_1$$

$$a_2 x + b_2 y = c_2$$

**y-column**

- Assign names for each matrix

**coefficient matrix:**

$$D = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$$

**X – matrix:**

$$D_x = \begin{bmatrix} c_1 & b_1 \\ c_2 & b_2 \end{bmatrix}$$

**Y – matrix:**

$$D_y = \begin{bmatrix} a_1 & c_1 \\ a_2 & c_2 \end{bmatrix}$$

$$x = \frac{D_x}{D} = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

$$y = \frac{D_y}{D} = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

**Example 1**: Solve the system with two variables by Cramer's Rule.

$$4x - 3y = 11$$

$$6x + 5y = 7$$

Start by extracting the three relevant matrices: coefficient, \large{x}$x$, and \large{y}$y$. Then solve each corresponding determinant.

- For **coefficient matrix**

$$D=\begin{bmatrix}4 & -3\\ 6 & 5\end{bmatrix}\xrightarrow{\text{find its determinant}}|D|=\begin{vmatrix}4 & -3\\ 6 & 5\end{vmatrix}=(4)(5)-(-3)(6)$$

$$=20-(-18)$$

$$=20+18$$

$$=38$$

- For **X – matrix**

$$D_x=\begin{bmatrix}11 & -3\\ 7 & 5\end{bmatrix}\xrightarrow{\text{find its determinant}}|D_x|=\begin{vmatrix}11 & -3\\ 7 & 5\end{vmatrix}=(11)(5)-(-3)(7)$$

$$=55-(-21)$$

$$=55+21$$

$$=76$$

- For **Y – matrix**

$$D_y=\begin{bmatrix}4 & 11\\ 6 & 7\end{bmatrix}\xrightarrow{\text{find its determinant}}|D_y|=\begin{vmatrix}4 & 11\\ 6 & 7\end{vmatrix}=(4)(7)-(11)(6)$$

$$=28-(66)$$

$$=-38$$

Once all three determinants are calculated, it's time to solve for the values of \large{x}$x$ and \large{y}$y$ using the formula above.

$$x = \frac{D_x}{D} = \frac{76}{38} = 2$$

$$y = \frac{D_y}{D} = \frac{-38}{38} = -1$$

I can write the final answer as \large{\left( {x,y} \right) = \left( {2, - 1} \right)}$(x,y)=(2,-1)$.

**Example 2**: Solve the system with two variables by Cramer's Rule.

$$3x + 5y = -7$$
$$x + 4y = -14$$

Setup your coefficient, \large{x}$x$, and \large{y}$y$ matrices from the given system of linear equations. Then calculate their determinants accordingly.
Remember that we **always subtract** the products of the diagonal entries.

- For the **coefficient matrix** (use the coefficients of both $x$ and $y$ variables)

$$D = \begin{bmatrix} 3 & 5 \\ 1 & 4 \end{bmatrix} \xrightarrow{\text{find its determinant}} |D| = \begin{vmatrix} 3 & 5 \\ 1 & 4 \end{vmatrix} = (3)(4) - (5)(1) = 7$$

- For the **X – matrix** (replace the x-column by the constant column)

$$D_x = \begin{bmatrix} -7 & 5 \\ -14 & 4 \end{bmatrix} \xrightarrow{\text{find its determinant}} |D_x| = \begin{vmatrix} -7 & 5 \\ -14 & 4 \end{vmatrix} = (-7)(4) - (5)(-14) = 42$$

- For the **Y – matrix** (replace the y-column by the constant column)

$$D_y = \begin{bmatrix} 3 & -7 \\ 1 & -14 \end{bmatrix} \xrightarrow{\text{find its determinant}} |D_y| = \begin{vmatrix} 3 & -7 \\ 1 & -14 \end{vmatrix} = (3)(-14) - (-7)(1) = -35$$

I hope you're getting comfortable computing for the determinant of a 2-dimensional matrix. To finally solve the required variables, I get the following results…

$$x = \frac{D_x}{D} = \frac{42}{7} = 6$$

$$y = \frac{D_y}{D} = \frac{-35}{7} = -5$$

Writing the final answer in point notation, I got \large{\left( {x,y} \right) = \left( {6, - 5} \right)}$(x,y)=(6,-5)$.