وزارة التعليم العالي والبحث العلمي
المعهد التقني/ السماوة
قسم المساحة

اسم المادة : Mathematics and Spherical Triangles

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(**EQUATION**)

EQUATION

An equation in mathematics is a statement that links 2 expression with the equal symbol.

One of the basic type of equations are the first degree equations , that can be called as "Linear equations".

Linear Equations with one variable:

A linear equation is an equation that represents a line and the general form of this equation is $ax + b = 0$ in which a and b are integers and x is the variable. This type of equation has only one solution and represents a parallel line with the y-axis.

HOW TO SOLVE LINEAR EQUATIONS WITH ONE VARIABLE?

Firstly let's write the basic Axioms that we apply when solving an equation:

1. Addition axiom: When two equal quantities are added on the both sides of an equation, the equation will still remain equal.

2. Subtraction axiom: When two equal quantities are subtracted on the both sides of an equation the equation will still remain equal.

3. Multiplication axiom: When we multiply the both sides of an equation with the same value, the equation will still remain equal.

4. Division axiom: When we divide both sides of an equation with the same value, the equation will still remain equal.

5. Distributive axiom: $a(b+c) = ab + ac$.

STEPS FOR SOLVING A LINEAR EQUATION WITH ONE VARIABLE

What we have to do is figure out how to isolate the variable x and for that the axioms above will help us. As a result, we will be using the axioms depending on the equation that we have.

1) Firstly we have to see which is the variable that we need to isolate.

2) Then distinguish the variables and the constants.

3) Group the variables on the left side and the constants on the right.

4) Using axioms that we listed above we perform algebraic operations so we can get the value of the variable.

Example 1: Solve $6x + 8 = 12$

$6x + 8 = 12$

$6x + 8 - 8 = 12 - 8$

$6x = 4$

$$\frac{6x}{6} = \frac{4}{6}$$

$$x = \frac{2}{3}$$

EXAMPLE 2: SOLVE $3(X+8) - 2 = 3(9-X)$

$3(X+8) - 2 = 3(9-X)$

$3X + 24 - 2 = 27 - 3X$

$3X + 22 = 27 - 3X$

$3X + 22 + 3X = 27 - 3X + 3X$

$6X + 22 = 27$

$6X + 22 - 22 = 27 - 22$

$6X = 5 \longrightarrow \frac{6x}{6} = \frac{5}{6} \longrightarrow x = \frac{5}{6}$

Example 3: Solve

$$\frac{6x+2}{3} - 1 = 3x$$

$$\frac{(6x+2)}{3} - \frac{3}{3} = 3x$$

$$(6x+2) - 3 = 9x$$

$$6x + 2 - 3 = 9x$$

$$6x - 9x = 1$$

$$-3x = 1$$

$$x = -\frac{1}{3}$$

Solve quadratic equations using the common law method

Common law is :

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Discriminant

Do you see $b^2 - 4ac$ in the formula above? It is called the Discriminant, because it can "discriminate" between the possible types of answer:

when $b^2 - 4ac$ is positive, we get two Real solutions

when it is zero we get just ONE real solution.

when it is negative we get a pair of Complex solutions

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Quadratic Equation in Standard Form: $ax^2 + bx + c = 0$

Using the Quadratic Formula

Just put the values of a, b and c into the Quadratic Formula, and do the calculations.

EXAMPLE: SOLVE $5X^2 + 6X + 1 = 0$

COEFFICIENTS ARE: $A = 5, B = 6, C = 1$

QUADRATIC FORMULA: $X = -B \pm \sqrt{(B^2 - 4AC)} \setminus 2A$

PUT IN A, B AND C: $X = -6 \pm \sqrt{(6^2 - 4\times5\times1)}/(2\times5)$

Solve: $x = -6 \pm \sqrt{(36 - 20)}/10$

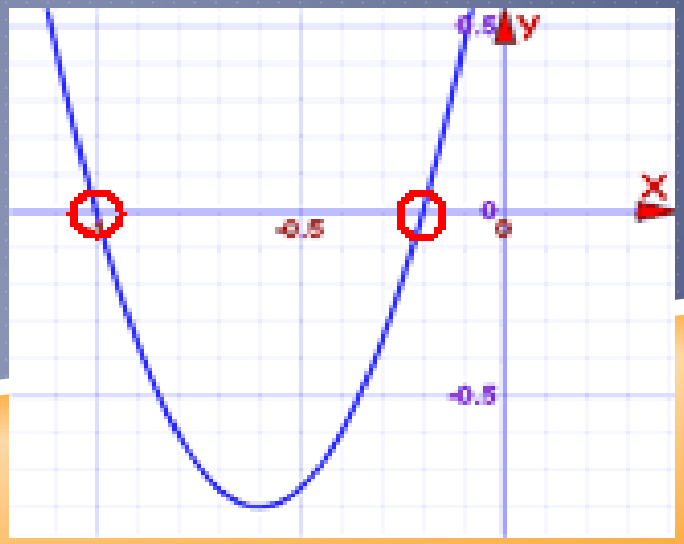
$x = -6 \pm \sqrt{(16)}/10$

$x = -6 \pm 4/10$

$x = -0.2$ or -1

Answer: $x = -0.2$ **or** $x = -1$

Check -0.2:		$5\times(-0.2)^2 + 6\times(-0.2) + 1$ $= 5\times(0.04) + 6\times(-0.2) + 1$ $= 0.2 - 1.2 + 1$ $= 0$
Check -1:		$5\times(-1)^2 + 6\times(-1) + 1$ $= 5\times(1) + 6\times(-1) + 1$ $= 5 - 6 + 1$ $= 0$



2) $X^2 - 4X - 5 = 0$

A = 1 , B = -4 , C = -5

$\Delta = B^2 - 4AC = (-4)^2 - 4x -5 = 16+20 = 36$

$$x = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-(-4) \pm \sqrt{36}}{2X1}$$

{-1,5} 4 ± 6 = 5

$$= \frac{4 \pm 6}{2}$$

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$$\therefore X_1 = \frac{4+6}{2} = \frac{10}{2} = 5$$

Or $$X_2 = \frac{4-6}{2} = \frac{-2}{2} = -1$$

Answer: $X_1 = 5$ **or** $X_2 = -1$

Complex Solutions?

When the Discriminant (the value $b^2 - 4ac$) is negative we get a pair of Complex solutions ... what does that mean?

It means our answer will include Imaginary Numbers. Wow

Example: Solve $5x^2 + 2x + 1 = 0$

Coefficients are: $a=5,\ b=2,\ c=1$

Note that the Discriminant is negative: $b^2 - 4ac = 2^2 - 4\times5\times1 = -16$

Use the Quadratic Formula : $x = -2 \pm \sqrt{(-16)}/10$

$\sqrt{(-16)} = 4i$

(where i is the imaginary number $\sqrt{-1}$)

So : $x = -2 \pm 4i/10$

Answer: $x = -2 \pm 0.4i$

we don't need more calculation, just leave it as $-0.2 \pm 0.4i$.

EXAMPLE: SOLVE $X^2 - 4X + 6.25 = 0$

Coefficients are: $a=1$, $b=-4$, $c=6.25$

Note that the Discriminant is negative: $b^2 - 4ac = (-4)^2 - 4\times1\times6.25$
$= -9$

Use the Quadratic Formula : $x = -(-4) \pm \sqrt{(-9)}/2$

$\sqrt{(-9)} = 3i$

(where i is the imaginary number $\sqrt{-1}$)

So : $x = 4 \pm 3i/2$

Answer: $x = 2 \pm 1.5i$

Summary

Quadratic Equation in Standard Form: $ax^2 + bx + c = 0$

Quadratic Formula: $x = -b \pm \sqrt{(b^2 - 4ac)}/2a$

When the Discriminant (b^2-4ac) is:

- positive, there are 2 real solutions
- zero, there is one real solution
- negative, there are 2 complex solutions

SOLVING A SYSTEM OF LINEAR EQUATIONS

SUBSTITUTION

$$\begin{cases} x + y = 3 \\ 2x - y = 0 \end{cases}$$

We isolate the *x* from the first equation:

:

$$x + y = 3 \rightarrow \quad x = 3 - y$$

And we substitute it in the second:

$$\begin{aligned} &2x - y = 0 \quad \rightarrow \\ &2(3 - y) - y = 0 \quad \rightarrow \\ &6 - 2y - y = 0 \quad \rightarrow \\ &6 - 3y = 0 \quad \rightarrow \\ &6 = 3y \quad \rightarrow \\ &y = \frac{6}{3} = 2 \end{aligned}$$

Knowing *y*, we obtain *x*:

$$x = 3 - y = 3 - 2 = 1$$

Therefore the solution to the system is

$$x = 1, \; y = 2$$

EQUALIZATION:

We isolate *y* from both equations:

$$x + y = 3 \quad \rightarrow \quad y = 3 - x$$
$$2x - y = 0 \quad \rightarrow \quad y = 2x$$

We equal both expressions and we resolve the equation:

$$3 - x = 2x \quad \rightarrow$$
$$3 = 3x \quad \rightarrow$$
$$x = \frac{3}{3} = 1$$

Substituting in the first of the previous equations

$$y = 3 - x = 3 - 1 = 2$$

Therefore the solution to the system is

,

$$x = 1, \; y = 2$$

SUBSTITUTION

$$\begin{cases} 5x - \frac{y}{2} = -1 \\ 3x - 2y = 1 \end{cases}$$

We isolate the *y* from the second equation:

$$3x - 2y = 1 \quad \rightarrow$$

$$-2y = 1 - 3x \quad \rightarrow$$

$$y = \frac{1 - 3x}{-2} = -\frac{1}{2} + \frac{3}{2}x$$

And we substitute it in the first equation:

$$5x - \frac{y}{2} = -1 \quad \rightarrow$$

$$5x - \frac{1}{2} \cdot \left(-\frac{1}{2} + \frac{3}{2}x\right) = -1 \quad \rightarrow$$

$$5x + \frac{1}{4} - \frac{3}{4}x = -1 \quad \rightarrow$$

$$4 \cdot 5x + 4 \cdot \frac{1}{4} - 4 \cdot \frac{3}{4}x = -4 \cdot 1 \quad \rightarrow$$

$$20x + 1 - 3x = -4 \quad \rightarrow$$

$$17x = -5 \quad \rightarrow \quad x = -\frac{5}{17}$$

Knowing *x*, we obtain *y*:

$$y = -\frac{1}{2} + \frac{3}{2}x =$$

$$= -\frac{1}{2} + \frac{3}{2}\left(-\frac{5}{17}\right) =$$

$$= -\frac{1}{2} - \frac{15}{34} = -\frac{16}{17}$$

THEREFORE THE SOLUTION TO THE SYSTEM IS

$$x = -\frac{5}{17}, \quad y = -\frac{16}{17}$$

Equalization

We isolate *y* from both equations:

$$5x - \frac{y}{2} = -1 \quad \rightarrow$$

$$-\frac{y}{2} = -1 - 5x \quad \rightarrow$$

$$y = -2(-1 - 5x) = 2 + 10x$$

$$3x - 2y = 1 \quad \rightarrow$$

$$-2y = 1 - 3x \quad \rightarrow$$

$$y = \frac{1 - 3x}{-2} = -\frac{1}{2} + \frac{3}{2}x$$

We equal both expressions and we resolve the equation:

$$2 + 10x = -\frac{1}{2} + \frac{3}{2}x \quad \rightarrow$$

$$2 \cdot 2 + 2 \cdot 10x = -2 \cdot \frac{1}{2} + 2 \cdot \frac{3}{2}x \quad \rightarrow$$

$$4 + 20x = -1 + 3x \quad \rightarrow$$

$$17x = -5 \quad \rightarrow$$

$$x = -\frac{5}{17}$$

SUBSTITUTING IN THE FIRST OF THE PREVIOUS EQUATIONS

$$y = 2 + 10x =$$
$$= 2 + 10\left(-\frac{5}{17}\right) =$$
$$= 2 - \frac{50}{17} = -\frac{16}{17}$$

We replace the value of *y* in the first equation and we solve it:

$$5x - \frac{y}{2} = -1 \rightarrow$$
$$5x - \frac{1}{2}\left(-\frac{16}{17}\right) = -1 \rightarrow$$
$$5x + \frac{8}{17} = -1 \rightarrow$$
$$x = \frac{-1 - \frac{8}{17}}{5} = -\frac{5}{17}$$

Then, the solution to the system is

$$x = -\frac{5}{17}, \quad y = -\frac{16}{17}$$

SOLVE SYSTEMS OF EQUATIONS GRAPHICALLY :

The geometric meaning of a linear system .

The general linear system consisting of two linear equations with tow x , y variables is represented by the following form

$$a_1x + b_1y = c_1$$

$$a_2x + b_2y = c_2$$

The geometry of these equations is straight lines and the point (x, y) lies on the line if and only if x , y satisfies the equation of the line .

There are three possibilities for solutions namely :

1 – The two lines are parallel , meaning there is no point of intersection , and therefore , the linear system has no solution .

2 – The two lines intersect with a point , which means that the linear system has only one solution .

3 – The two lines are congruent , meaning there are an unlimited number of solutions .

SOLVING LINEAR EQUATIONS USING GRAPHS

We represent the first equation and then represent the second equation, the point of confluence of those lines that gives the solution to the system of equation .

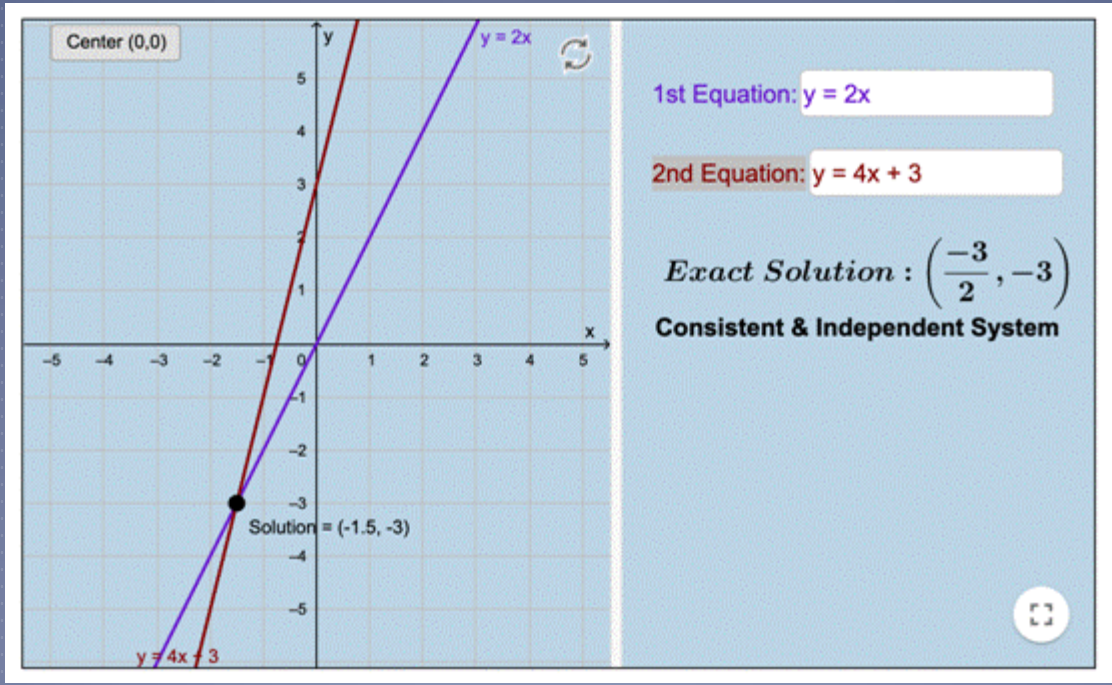
Whenever we change the representation of these lines according to the existing system we have , the solution changes to give the new system solution.

system. For example , if you are asked to find a solution to the following

$y=2x$, $y=4x+3$

We represent the first line that does not lie on the y axis at any point because there is no fixed coefficient of the equation and we choose two (0,0) and the point points to represent the line , for example the point (1,2) and we represent it , and likewise the other line , we choose two (points (0,3),(1,7)).

AND WE REPRESENT IT SO THAT THE RESULT OF THE INTERSECTION OF THE TOW LINES IS THE SOLUTION TO THE SYSTEM



Homework

EXAMPLE : SOLVE THE FOLLOWING LINEAR EQUATION

$$\begin{cases} -10x - 5y = 0 \\ 21x - 7y = 28 \end{cases}$$