وزارة التعليم العالي والبحث العلمي
المعهد التقني/ السماوة
قسم المساحة

اسم المادة : Mathematics and Spherical Triangles

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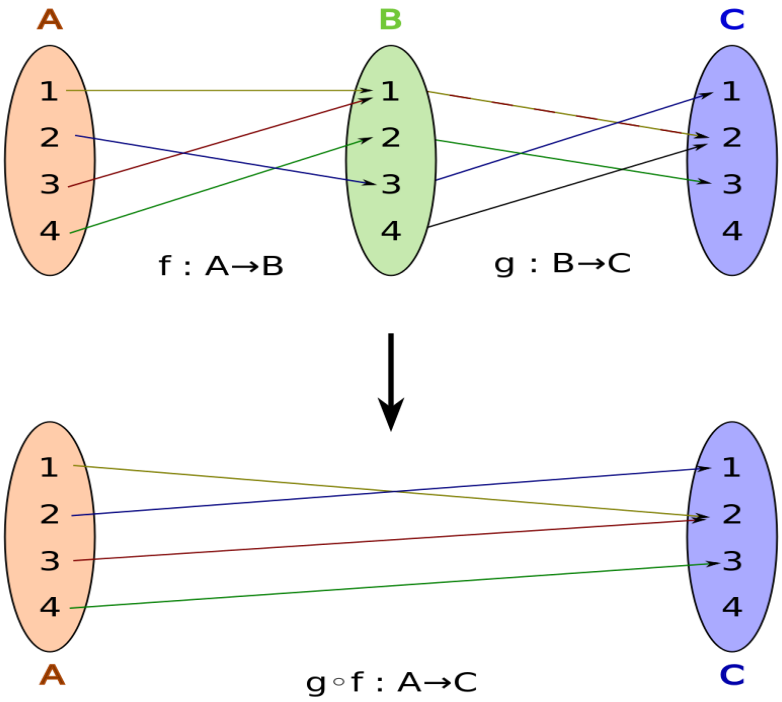
- Function
- The derivative as a function
- Derivative as a polynomial function

Function

In mathematics, a function is a binary relation between two sets that associates to each element of the first set exactly one element of the second set. Typical examples are functions from integers to integers, or from the real numbers to real numbers.



A polynomial function

A polynomial function is a function that can be expressed in the form of a polynomial. The definition can be derived from the definition of a polynomial equation. A polynomial is generally represented as P(x). The highest power of the variable of P(x) is known as its degree. Degree of a polynomial function is very important as it tells us about the behaviour of

THE DERIVATIVE AS A FUNCTION

If a given function f is differentiable at a domain value a, then f '(a) is a real number. In this way f ' itself becomes a new function, called the **derivative** of f. The domain of the derivative is the set of all points in the domain of f at which f itself is differentiable.

the function P(x) when x becomes very large. The domain of a polynomial function is entire real numbers (R).

If $P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots\ldots\ldots\ldots + a_2 x^2 + a_1 x + a_0$.

Derivative of a polynomial function

The derivative of a polynomial function involving multiple linear and/or power functions can be found using the formulas for finding linear and power functions, along with the constant multiple and sum rules.

1 - If f is a constant function, where $f(x) = c$, then

$$(c)' = 0 \qquad \frac{d}{dx}(c) = 0$$

2- If f is a power function, where $f(x) = x^n$ and n is a real number, then

$$f'(x) = nx^{n-1} \qquad \frac{d}{dx}(x^n) = nx^{n-1}$$

3 - If f and g are differentiable functions, then

$$[f(x) + g(x)]' = f'(x) + g'(x)$$

4 - If f and g are differentiable functions, then

$$[f(x) - g(x)]' = f'(x) - g'(x)$$

5 -If f and g are differentiable functions, then

$$[f(x)g(x)]' = g(x)f'(x) + f(x)g'(x)$$

6 - If f and g are differentiable functions, then

$$\left[\frac{f(x)}{g(x)}\right]' = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

Example : If f(x) = 2 for all x in **R**, then f '(2) = 0.

Example : If f(x) = 2x for all x in **R**, then f '(x) = 2.

Example : If f(x) = x^2 for all x in **R**, then f '(x) = 2x.

Example : Find the derivative of the following function.

$$\begin{aligned}\frac{d}{dx}\left[(2x-1)\left(3x^2+2\right)\right] &= \frac{d}{dx}\left(6x^3-3x^2+4x-2\right)\\ &= 18x^2-6x+4.\end{aligned}$$

Example : Find the derivative of the following function.

$$\begin{aligned}\frac{d}{dx}\left(5x^6+3x^4-8x^2+1\right) &= 5\left(6x^5\right)+3\left(4x^3\right)-8\left(2x\right)\\ &= 30x^5+12x^3-16x.\end{aligned}$$

Example : Find the derivative of the following function.

$$s = 13t^3 - 11t^2 + 25$$

$$\frac{ds}{dt} = 39t^2 - 22t.$$

Example : Find the derivative of the following functions.

$$a) \quad f(x) = \pi x = \pi x^1$$
$$f'(x) = \pi(1)x^{1-1}$$
$$= \pi x^0$$
$$= \pi(1)$$
$$= \pi$$

$\therefore \quad f'(x) = \pi.$ This derivative could have also been found using the constant rule.

$$b) \quad g(y) = 5y^3$$
$$g'(y) = 5(3)y^{3-1}$$
$$= 15y^2$$

$$\therefore \quad g'(y) = 15y^2.$$

$$c) \quad y = -\frac{8}{x^2} = -8x^{-2}$$
$$y' = -8(-2)x^{-2-1}$$
$$= 16x^{-3}$$
$$= \frac{16}{x^3}$$

$$\therefore \quad y' = \frac{16}{x^3}.$$

$$d) \quad v(t) = \sqrt{32}t^{\sqrt{2}}$$
$$= \left(\sqrt{32}\right)\left(\sqrt{2}\right)t^{\sqrt{2}-1}$$
$$= \left(\sqrt{(32)(2)}\right)t^{\sqrt{2}-1}$$
$$= \left(\sqrt{64}\right)t^{\sqrt{2}-1}$$
$$= 8t^{\sqrt{2}-1}$$

$$\therefore \quad v(t) = 8t^{\sqrt{2}-1}.$$

Implicit Derivative

The derivative that is found by using the process of implicit differentiation is called the implicit derivative. For example, the derivative $\frac{dy}{dx}$ and it is called the implicit derivative. An implicit derivative usually is in terms of both x and y.

How to do Implicit Differentiation

- Differentiate with respect to x.
- Collect all the $\frac{dy}{dx}$ on one side.
- Solve for $\frac{dy}{dx}$.

Example: Find the derivative of the following function.

$x^2 + y^2 = a^2$

Solution:

Differentiate with respect to x:

$$\frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) = (a^2)$$

$$2x + 2y\ \frac{dy}{dx} = 0$$

Collect all the $\frac{dy}{dx}$ on one side

$$y\ \frac{dy}{dx} = -x$$

Solve for $\frac{dy}{dx}$:

$$\frac{dy}{dx} = \frac{-x}{y}$$

Example : Find the derivative of the following function.

$10x^4 - 18xy^2 + 10y^3 = 48$

Solution:

Start with: $10x^4 - 18xy^2 + 10y^3 = 48$

Derivative: $10(4x^3) - 18(x(2y\frac{dy}{dx}) + y^2) + 10(3y^2\frac{dy}{dx}) = 0$

Simplify: $40x^3 - 36xy\frac{dy}{dx} - 18y^2 + 30y^2\frac{dy}{dx} = 0$

$$-36xy\frac{dy}{dx} + 30y^2\frac{dy}{dx} = -40x^3 + 18y^2$$

Simplify : $(30y^2 - 36xy)\frac{dy}{dx} = 18y^2 - 40x^3$

Simplify : $(15y^2 - 18xy)\frac{dy}{dx} = 9y^2 - 20x^3$

And we get:

$$\frac{dy}{dx} = 9y^2 - 20x^3 / \ 3(15y^2 - 18xy)$$

Example : Find the derivative of the following function.

$x^3y^5 + 3x = 8y^3 + 1$

Solution:

$x^3y^5 + 3x = 8y^3 + 1$

$$3x^2y^5 + 5x^3y^4y' + 3 = 24y^2y'$$

$$3x^2y^5 + 3 = 24y^2y' - 5x^3y^4y'$$

$$3x^2y^5 + 3 = y' (24y^2 - 5x^3y^4)$$

$$y' = (3x^2y^5 + 3)/(24y^2 - 5x^3y^4)$$