وزارة التعليم العالي والبحث العلمي
المعهد التقني/ السماوة
قسم المساحة

اسم المادة : Mathematics and Spherical Triangles

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- Multiplication of Matrices
- Transposing

Multiplication of Matrices

Multiply by a Constant

We can multiply a matrix by a **constant** *(the value 2 in this case)*:

$$2 \times \begin{bmatrix} 4 & 0 \\ 1 & -9 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 2 & -18 \end{bmatrix}$$

2×4=8

These are the calculations:

2×4=8 2×0=0

2×1=2 2×−9=−18

Multiplying by Another Matrix

$$\begin{array}{c} \vec{a_1} \rightarrow \\ \vec{a_2} \rightarrow \end{array} \begin{bmatrix} 1 & 7 \\ 2 & 4 \end{bmatrix} \cdot \begin{array}{c} \begin{array}{cc} \vec{b_1} & \vec{b_2} \\ \downarrow & \downarrow \end{array} \\ \begin{bmatrix} 3 & 3 \\ 5 & 2 \end{bmatrix} \end{array} = \begin{bmatrix} \vec{a_1} \cdot \vec{b_1} & \vec{a_1} \cdot \vec{b_2} \\ \vec{a_2} \cdot \vec{b_1} & \vec{a_2} \cdot \vec{b_2} \end{bmatrix}$$

$$A \qquad B \qquad C$$

Matrix multiplication

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 5 & 6 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 1*5 + 2*0 & 1*6 + 2*7 \\ 3*5 + 4*0 & 3*6 + 4*7 \end{bmatrix} = \begin{bmatrix} 5 & 20 \\ 15 & 46 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \text{X} \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 10 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix} \text{X} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 7 & 10 \end{bmatrix}$$

Matrix Multiplication

A **B** **A * B**

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \begin{pmatrix} 6 & 3 \\ 5 & 2 \\ 4 & 1 \end{pmatrix} = \begin{pmatrix} 1*6 + 2*5 + 3*4 & 1*3 + 2*2 + 3*1 \\ 4*6 + 5*5 + 6*4 & 4*3 + 5*2 + 6*1 \end{pmatrix}$$

For $\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 5 & 4 \\ -5 & 1 \end{bmatrix}$

$$\mathbf{AB} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 5 & 4 \\ -5 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 5 & 9 \end{bmatrix}$$

$$\mathbf{BA} = \begin{bmatrix} 5 & 4 \\ -5 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 13 & 4 \\ -3 & 1 \end{bmatrix}$$

Transposing

the transpose of a matrix is an operator which flips a matrix over its diagonal; that is, it switches the row and column indices of the matrix A by producing another matrix, often denoted by A^T (among other notations)

1. **Start with any matrix. You can transpose any matrix, regardless of how many rows and columns it has.**
2. **Turn the first row of the matrix into the first column of its transpose. ...**
3. **Repeat for the remaining rows. The second row of the original matrix becomes the second column of its transpose**.

Transpose of a Matrix

$$\begin{bmatrix} 3 & 4 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}$$

Input Matrix — Transpose Matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2\times 3} \qquad \mathbf{A}^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}_{3\times 2}$$

- $(\mathbf{A}^T)^T = A$
- $(\mathbf{A} + \mathbf{B})^T = \mathbf{A}^T + \mathbf{B}^T$
- $(\mathbf{A}\,\mathbf{B})^T = \mathbf{B}^T\mathbf{A}^T$
- $(c \cdot \mathbf{A})^T = c \cdot \mathbf{A}^T$

To "transpose" a matrix, swap the rows and columns.

We put a "T" in the top right-hand corner to mean transpose:

$$\begin{bmatrix} 6 & 4 & 24 \\ 1 & -9 & 8 \end{bmatrix}^T = \begin{bmatrix} 6 & 1 \\ 4 & -9 \\ 24 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}^T = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

Transpose of a matrix

$$\begin{pmatrix} 5 & 4 \\ 4 & 0 \\ 7 & 10 \\ -1 & 8 \end{pmatrix}_{4 \times 2}^T = \begin{pmatrix} 5 & 4 & 7 & -1 \\ 4 & 0 & 10 & 8 \end{pmatrix}_{2 \times 4}$$

$$\mathbf{A} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}, \mathbf{A}^T = \begin{bmatrix} 2 & 3 & -1 \end{bmatrix} \qquad \mathbf{B} = \begin{pmatrix} 1 & 2 & 4 \\ 0 & -3 & 1 \end{pmatrix}, \mathbf{B}^T = \begin{pmatrix} 1 & 0 \\ 2 & -3 \\ 4 & 1 \end{pmatrix}$$

$$\mathbf{C} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, \mathbf{C}^T = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} \qquad \mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{I}^T = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}^{T} = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

Transpose of a matrix

$$\text{If } \mathbf{A} = \begin{bmatrix} 1 & 5 & 9 \\ 2 & 3 & 8 \\ 6 & 8 & 7 \end{bmatrix}, \text{ then } \mathbf{A}^T = \begin{bmatrix} 1 & 2 & 6 \\ 5 & 3 & 8 \\ 9 & 8 & 7 \end{bmatrix}$$

Matrix : Transpose of matrix :

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

Matrix : Transpose of matrix :

$$\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

$$M = \begin{bmatrix} 3 & -4 \\ 2 & 0 \\ -1 & 6 \\ 5 & 1 \end{bmatrix}$$

$$M^T = \begin{bmatrix} 3 & -4 \\ 2 & 0 \\ -1 & 6 \\ 5 & 1 \end{bmatrix}^T = \begin{bmatrix} 3 & 2 & -1 & 5 \\ -4 & 0 & 6 & 1 \end{bmatrix}$$