جامعة الفرات الاوسط التقنية / المعهد التقني- سماوة
قسم التقنيات الكهربائية

المحاضرة العاشرة

اسم المادة / دوائر كهربائية

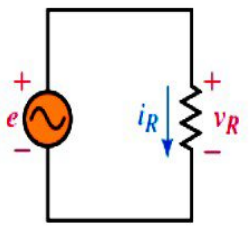
المرحلة الدراسية / الاولى

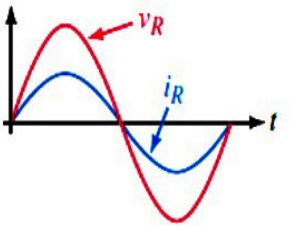
العام الدراسي / 2022 – 2023

استاذ المادة : بشائر مكي جبر

AC circuit with pure resistance:

A circuit without inductance and capacitance is called pure resistance circuit as shown in figure.





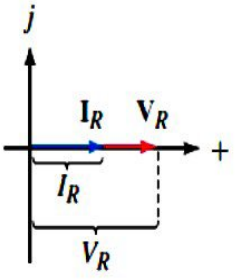
By applying Ohm's law:

$$i_R = \frac{v_R}{R} = \frac{V_m \sin \omega t}{R} = \frac{V_m}{R} \sin \omega t = I_m \sin \omega t$$

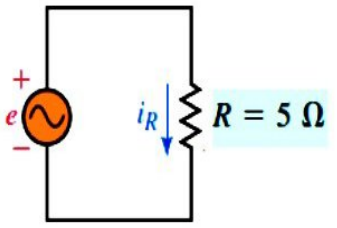
where $I_m = V_m/R$

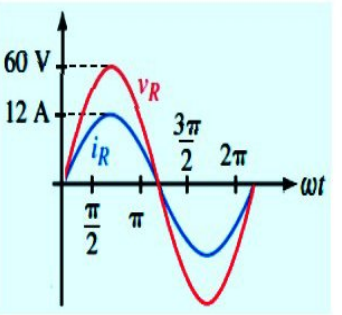
Transposing, $V_m = I_m R$

Note that current and voltage as in phase:



Example-21: For the circuit shown in figure find the value of i_R , if v(t)=60sinωt.



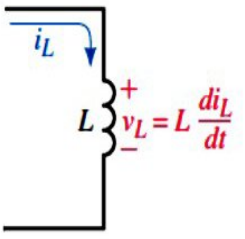


Solution:

$$i_R = \frac{v}{R} = \frac{60\sin(\omega t)}{5} = 12\sin(\omega t) \quad A$$

AC circuit with Purely Inductive Load:

Consider a purely inductive circuit with an inductor connected to an AC generator, as shown in Figure.



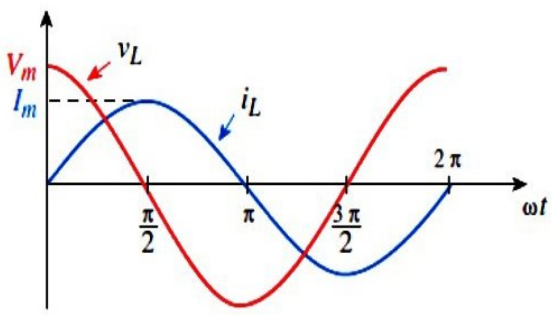
$$v_L = L\frac{di_L}{dt} = L\frac{d}{dt}(I_m \sin\omega t) = \omega L I_m \cos\omega t = V_m \cos\omega t$$

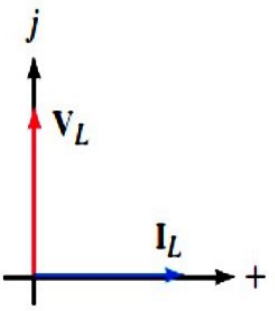
Utilizing the trigonometric identity [$\cos\omega t = \sin(\omega t + 90^\circ)$ you can write this as:

$$v_L = V_m \sin(\omega t + 90°)$$

$$V_m = \omega L I_m$$

Voltage and current waveforms are shown in Figure (a), and phasors in Figure(b). As you can see, *for a purely inductive circuit, current lags voltage by 90°*





From Equation above, we see that the ratio V_m to I_m is:

$$\frac{V_m}{I_m} = \omega L$$

This ratio is defined as **inductive reactance** and is given the symbol *XL*. Since the ratio of volts to amps is ohms, reactance has units of ohms.

$$X_L = \frac{V_m}{I_m} \quad (\Omega)$$

Combining Equations above yields:

$$X_L = \omega L \quad (\Omega)$$

Where ω is radians per second $\omega = 2\pi f$

Example-22: The voltage across a 0.2H inductance is $v_L = 100 \sin(400t - 70°)$ V. Determine i_L and sketch it

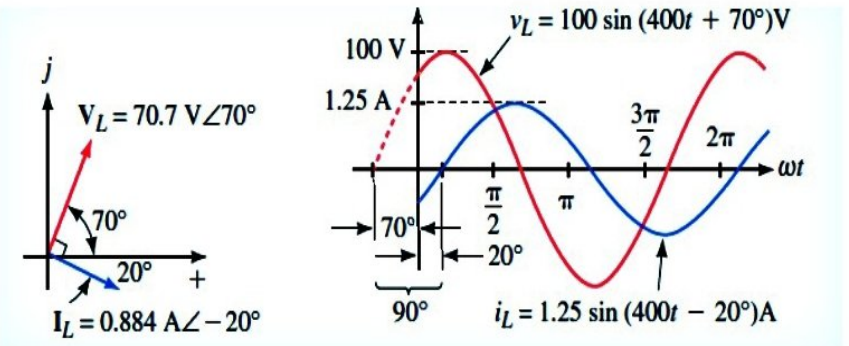
Solution:

ω = 400 rad/s therefore $x_L = \omega L = 400 \times 0.2 = 80\Omega$

$$I_m = \frac{V_m}{x_L} = \frac{100}{80} = 1.25A$$

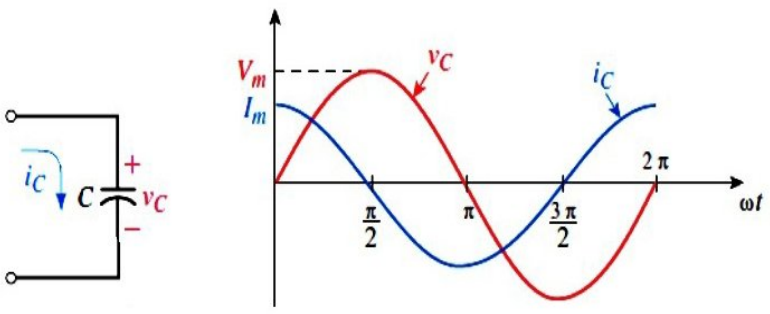
The current lags the voltage by 90° therefore:

$i_L = 1.25\sin(400t - 20^{\circ})$ A as indicated in figure below



AC circuit with Purely Capacitive Load:

In the purely capacitive case, both resistance R and inductance L are zero. The circuit diagram is shown in Figure



For capacitance, current is proportional to the rate of change of voltage:

$$i_C = C\frac{dv_C}{dt} = C\frac{d}{dt}(V_m \sin \omega t) = \omega C V_m \cos \omega t = I_m \cos \omega t$$

Using the appropriate trigonometric identity, this can be written as:

$$i_C = I_m \sin(\omega t + 90°)$$

Where $I_m = \omega C V_m = \frac{V_m}{X_c}$

$\mathbf{x_c}$ is called the capacitance reactance

<u>Example-23</u>: The voltage across a 10-mF capacitance is $v_C = 100\sin(\omega t - 40°)$ V and f = 1000 Hz. Determine i_C and sketch its waveform.

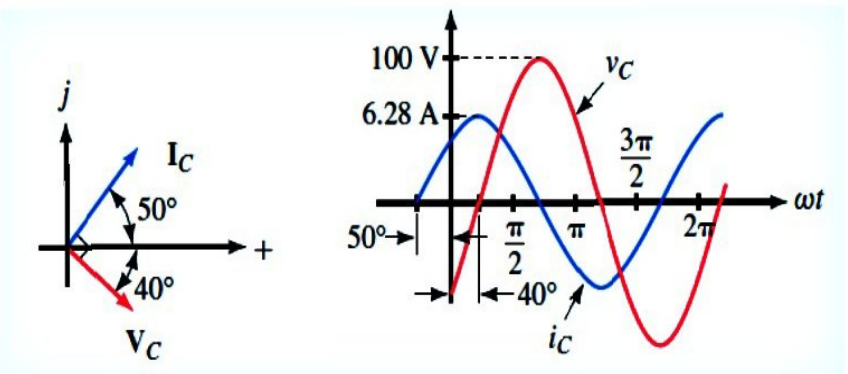
Solution:

$$\omega = 2\pi f = 2\pi(1000 \text{ Hz}) = 6283 \text{ rad/s}$$

$$X_C = \frac{1}{\omega C} = \frac{1}{(6283)(10 \times 10^{-6})} = 15.92\ \Omega$$

$$I_m = \frac{V_m}{X_C} = \frac{100 \text{ V}}{15.92\ \Omega} = 6.28 \text{ A}$$

Since current leads voltage by 90°, i_C = 6.28 sin(6283t - 50°) A as indicated in Figure below.



Impedance:

The opposition that a circuit element presents to current in the phasor domain is defined as its **impedance.**

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} \quad \text{(ohms)}$$

For resistor:

$$\mathbf{Z}_R = R$$

For inductance:

$$\mathbf{Z}_L = j\omega L = jX_L$$

For capacitance:

$$\mathbf{Z}_C = -j\frac{1}{\omega C} = -jX_C \quad \text{(ohms)}$$